EXPLAINING CROSS-COHORT DIFFERENCES IN LIFE CYCLE EARNINGS

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The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.
Introduction
Earnings profiles are becoming flatter

Source: IPUMS; Data: Synthetic cohorts of white men, employed and working for a wage.
Earnings profiles are becoming flatter

- Earnings growth for a cohort:
  \[
  \frac{\text{Earnings per worker at age 55}}{\text{Earnings per worker at age 25}}
  \]

- Earnings growth for 1940 cohort: 3.9; Earnings growth for 1980 cohort: 2.2
  \[
  \frac{1980\text{-cohort earnings growth}}{1940\text{-cohort earnings growth}} = 0.57
  \]

- The life cycle earnings profile flattened by 43% between the two cohorts.
Another view of flattening

- Age 25: Earnings in 1940 and 1980
- Age 55: Earnings in 1970 and 2010

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<tr>
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This paper

• The Model
  • College choice
  • Human capital accumulation on the job, à la Ben-Porath
  • Heterogenous ability to accumulate human capital
  • Only exogenous difference between cohorts: productivity
**The Model**
- College choice
- Human capital accumulation on the job, à la Ben-Porath
- Heterogenous ability to accumulate human capital
- Only exogenous difference between cohorts: productivity

**The Mechanism**
- Productivity growth $\Rightarrow$ recent cohort starts with higher productivity $\Rightarrow$ recent cohort has higher college enrollment
- Avg. ability of college- & hs-worker ↓ $\Rightarrow$ flatter earnings
This paper

- The Model
  - College choice
  - Human capital accumulation on the job, à la Ben-Porath
  - Heterogeneous ability to accumulate human capital
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- The Mechanism
  - Productivity growth \( \Rightarrow \) recent cohort starts with higher productivity \( \Rightarrow \) recent cohort has higher college enrollment
  - Avg. ability of college- & hs-worker ↓ \( \Rightarrow \) flatter earnings

- The Exercise
  - Calibrate to enrollment time series, 1940-cohort earnings
The Model
Environment

- Age \( j = 1, \ldots, J \),
- Ability \( a > 0 \), CDF \( A \)
  - Ability \( a \) constant over life
  - CDF \( A \) identical across cohorts
- Perfect foresight, perfect credit market, gross rate of interest \( r \)
- Exogenous productivity: \( w \) (growth \( g \) )
- Age 1: HS education, human capital \( h_1(a) = z_H a \)
- College takes \( s \) years
Environment

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- College takes \( s \) years

Decisions:
- Attend college \( \in \{\text{yes, no}\} \)
- Time in human capital accumulation on the job \( \in (0, 1] \)
**Human Capital Accumulation on the Job**

- Age $j$, human capital $h$, productivity $w$, ability $a$:

  $$W_j(h, w, a) = \max_{n \in (0,1]} \left[ wh(1 - n) + \frac{1}{r} W_{j+1}(h', w, a) \right]$$

  s.t.

  $$h' = (1 - \delta)h + F(nh, a)$$

  $$W_{J+1} = 0$$

- $F(nh, a) = z_F a(nh)^{\phi}$

- $\delta$: depreciation rate
Human Capital Accumulation on the Job

- Value function at age $j$:
  \[ W_j(h, w, a) = \beta_j(w)h + \alpha_j(w, a) \]
- $\beta_j(w)$, marginal return to human capital for worker of age $j$
  - Proportional to $w$
Earnings Growth

- First order condition for worker of age $j$:

$$w = \frac{1}{r} \beta_{j+1}(wg)F_1(nh, a) \Rightarrow F_1(nh, a) = \text{constant}_j$$

- $nh$ independent of $w$
- Productivity growth $g$ constant across cohorts
- $\Rightarrow$ $nh$ at age $j$ constant across cohorts
**Earnings Growth**

- First order condition for worker of age $j$:
  \[
  w = \frac{1}{r} \beta_{j+1}(wg) F_1(nh, a) \Rightarrow F_1(nh, a) = \text{constant}_j
  \]

  - $nh$ independent of $w$
  - Productivity growth $g$ constant across cohorts
  - $\Rightarrow nh$ at age $j$ constant across cohorts

- Earnings: $wh(1 - n) = wh - wnh$
  - Cross-cohort differences in age-$j$ earnings due to $w$ and $h$
  - Cross-cohort differences in age-$j$ earnings growth only due to $h$
Human Capital Accumulation on the Job

• Growth of $h$ within cohort:

$$\frac{h_{j+1}}{h_j} = 1 - \delta + \frac{F((nh)_j, a)}{h_j}$$

• Higher $a \Rightarrow$ steeper earnings profiles

• Higher $h \Rightarrow$ flatter earnings profiles
Human capital at the start of work life

- High school workers: Exogenous human capital \( h_1(a) = z_H a \)
- College workers: Endogenous college human capital
  \[
  G(k, h_1(a), a) = (z_G k)^\eta (ah_1(a))^{1-\eta}
  \]
- \( k \): present value of goods spending in college
- First order condition for \( k \):
  \[
  1 = \frac{1}{\rho s} \beta_{s+1} (wg^s) G_1 (k^*, h_1(a), a)
  \]
- College human capital increasing in \( a \) and \( w \)
- College human capital higher for recent cohorts
Schooling

- The schooling choice

\[
\max \left\{ V^{hs}(a, w), V^{col}(a, w) \right\} = \\
\max \left\{ W_1(z_Ha, w, a), W_{s+1}(G(k^*, z_Ha, a), w^s, a) - k^* \right\}
\]

- Marginal worker

\[
V^{hs}(a^*, w) = V^{col}(a^*, w)
\]

- Higher productivity \(\Rightarrow\) lower goods cost of college \(\Rightarrow\) higher college enrollment, i.e. \(da^*/dw < 0\)
The Composition Effect

A'(a)

Top hs. workers become bottom col. workers
Analysis
**Earnings Growth: the old cohort**

- Higher ability $\Rightarrow$ higher earnings growth
- Higher ability $\Rightarrow$ higher human capital $\Rightarrow$ lower earnings growth
Earnings Growth: cross-cohort differences

- More college workers ⇒ average ability ↓
- Fewer high school workers ⇒ average ability ↓
- Given $a$, lower earnings growth for college workers
The Experiment
CALIBRATION

- Model period = 1 year, \( J = 50 \)
- \( s = 4, \ r = 1.05, \ w_{1940} = 1, \ \delta = 1.14\% \)

- Remaining parameters \( \theta = (\mu, \sigma, \eta, z_H, z_G, z_F, g, \phi)' \):
  - Minimize distance to targets
    - Earnings growth for hs. worker, by age, 1940 cohort
    - Earnings growth for col. worker, by age, 1940 cohort
    - Earnings dispersion (cv) for hs. worker, by age, 1940 cohort
    - Earnings dispersion (cv) for col. worker, by age, 1940 cohort
    - Time series of col. attainment by cohort

# Calibration

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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Ability distribution</td>
<td>$\mu = -0.91$, $\sigma = 0.34$</td>
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<tr>
<td>Initial human capital</td>
<td>$z_H = 1.40$</td>
</tr>
<tr>
<td>College technology</td>
<td>$\eta = 0.45$, $z_G = 1.16$</td>
</tr>
<tr>
<td>On-the-job technology</td>
<td>$\phi = 0.66$, $z_F = 0.18$</td>
</tr>
<tr>
<td>Wage per human capital</td>
<td>$g = 1.005$, $w_{1940} = 1.0$</td>
</tr>
<tr>
<td>Life expectancy, college length</td>
<td>$J = 50$, $s = 4$</td>
</tr>
<tr>
<td>Interest rate, depreciation</td>
<td>$r = 1.05$, $\delta = 0.0114$</td>
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## Calibrated Moments

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<td>0.18</td>
<td>0.61</td>
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</tr>
<tr>
<td>CV at 45</td>
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<td>0.47</td>
<td>0.61</td>
<td>0.55</td>
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<tr>
<td>CV at 55</td>
<td>0.45</td>
<td>0.53</td>
<td>0.65</td>
<td>0.62</td>
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Educational Attainment: Model v. Data

The graph illustrates the percentage of individuals with a college degree as a function of the year of their 25th birthday for both the model and data. The x-axis represents the year of the 25th birthday, ranging from 1940 to 1980, and the y-axis shows the percentage of individuals with a college degree, ranging from 0.35 to 0.55. The model is represented by a solid blue line, while the data is indicated by red circles.

Key points:
- In 1940, the model predicts 0.35% with a college degree, whereas the data shows a slightly lower percentage.
- By 1980, both the model and data show an increase, with the model predicting 0.55% and the data showing a similar trend.

The graph captures the trend of increasing educational attainment over time, with the model closely following the data.
Main Experiment

- Let \( w \) grow at rate \( g \), compute decisions of cohorts

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<tr>
<td>Ratio of earnings growth (age 25 to 55)</td>
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<td></td>
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</tr>
<tr>
<td>Data</td>
<td>0.57</td>
<td>0.41</td>
<td>0.66</td>
</tr>
<tr>
<td>Model</td>
<td>0.73</td>
<td>0.76</td>
<td>0.72</td>
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Flattening from the 1940 cohort to the 1980 cohort (percent)

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<td>59</td>
<td>34</td>
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<tr>
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<td>27</td>
<td>24</td>
<td>28</td>
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<td>Model / Data</td>
<td>63</td>
<td>42</td>
<td>82</td>
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Decomposition

- Avg. high school educated worker of cohort $x$: ability $\bar{a}_x$ with initial human capital $h_x$
- $\gamma(a, h) \equiv$ earnings growth (earnings at 55 / 25)
- Earnings growth for the avg. high school educated worker: $\gamma(\bar{a}_{80}, h_{80}) < \gamma(\bar{a}_{40}, h_{40})$ since

$$\bar{a}_{80} < \bar{a}_{40} \Rightarrow h_{80} < h_{40}$$

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**DECOMPOSITION**

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Cross-Cohort Differences in Productivity Growth

- Productivity growth $g = 1.005$ for all cohorts in baseline
- What if $g$ is lower for the 1980 cohort?
**Cross-Cohort Differences in Productivity Growth**

- Productivity growth $g = 1.005$ for all cohorts in baseline
- What if $g$ is lower for the 1980 cohort?
  - Let $g = 1.0025$ for 1980 cohort, re-calibrate
  - Accounts for
    - 70% of flattening for all workers (vs. 63%)
    - 51% of flattening for hs workers (vs. 42%)
    - 89% of flattening for college workers (vs. 82%)
Cross-Cohort Differences in Ability Distribution

- $A$ may differ across cohorts (Hendricks and Schoellman, 2014)
- Alternative density for 1980 cohort: $\zeta A'(a)B_\lambda(a)$
- $B_\lambda$: density of exponential with $\lambda$
- $\zeta \int A'(a)B_\lambda(a) = 1$

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<td>Baseline</td>
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<tr>
<td>$\lambda = 1$</td>
<td>70 43 93</td>
</tr>
<tr>
<td>$\lambda = 2$</td>
<td>75 45 101</td>
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See distributions.
Cross-Cohort Differences in Ability Distribution

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Conclusion
CONCLUDING REMARKS

• Earning profiles flatter for recent cohorts

• Why?
  • Increasing productivity

• How?
  • Productivity growth ⇒ recent cohort starts with higher productivity ⇒ lower goods cost of college ⇒ recent cohort has higher college enrollment
  • Avg. ability of college- & hs-worker ↓ ⇒ flatter earnings

• Calibrated model accounts for 63% of the flattening in earnings profiles between 1940 and 1980 cohorts
Extra Material
Human Capital Accumulation on the Job

Solution:

\[ W_j(h, w, a) = \beta_j(w)h + \alpha_j(w, a) \]
\[ \beta_j(w) = w + \frac{1 - \delta}{r} \beta_{j+1}(wg) \]
\[ \alpha_j(w) = \kappa w \left[ \frac{1}{r} \frac{\beta_{j+1}(w \tau g)}{w} z_F a \right]^{1/(1-\phi)} + \frac{1}{r} \alpha_{j+1}(wg, a) \]
\[ \beta_j(w) = w \]
\[ \alpha_j(w, a) = 0 \]
Human Capital Accumulation on the Job

- Corner solution: $n = 1$

$$W_j(h, w, a) = \frac{1}{r} W_{j+1}(h', w_g, a)$$

s.t. $h' = (1 - \delta)h + F(1, h, a)$
**Effect of Productivity on Educational Attainment**

Present value of earnings

College attainment: old cohort

College attainment: recent cohort

\[ a^*(w_{\text{recent}}) \]

\[ a^*(w_{\text{old}}) \]
**Distance to data**

\[
\begin{align*}
\min_{\theta} & \sum_{i \in \{hs, co\}} \sum_{j=35, 45, 55} \left( \frac{E^i_{1940,j}(\theta)/E^i_{1940,25}(\theta)}{E^i_{1940,j}/E^i_{1940,25}} - 1 \right)^2 \\
& + \sum_{i \in \{hs, co\}} \sum_{j=25, 35, 45, 55} \left( \frac{S^i_{1940,j}(\theta)/E^i_{1940,j}(\theta)}{S^i_{1940,j}/E^i_{1940,j}} - 1 \right)^2 \\
& + \sum_{\tau=1940, 1950, \ldots, 1980} (p^\tau(\theta)/p^\tau - 1)^2
\end{align*}
\]
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Educational Attainment: Model v. Data

![Graph showing the comparison of model and data for educational attainment with college degrees over years. The x-axis represents the year of 25th birthday, ranging from 1940 to 1980, and the y-axis represents the percentage with college degree, ranging from 0.35 to 0.55. The blue line represents the model, and the red line represents the data. The graph shows a trend of increasing educational attainment with time.](image-url)
The Composition Effect

Top hs. workers become bottom col. workers

$A'(a)$

$a^*(w_{recent})$  $a^*(w_{old})$
**Productivity and Cohort Lives**
Alternative Distributions of Types for 1980 Cohort
Another view of flattening

- Age 25: Earnings in 1940 and 1980
- Age 55: Earnings in 1970 and 2010

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EARNINGS INEQUALITY: TIME SERIES

Earnings at 55/25

Year


Actual
Assuming 1940-cohort growth
Earnings Growth for Average Worker

- Selection effect explains flattening for HS. and Col. workers... but average worker has the same ability in all cohorts since the ability distribution is the same in all cohorts.
- What explains the flattening for average worker?
- Higher college enrollment $\Rightarrow$ Average worker has higher human capital: 

$$h = \text{col. frac} \times h_{col} + (1 - \text{col. frac}) \times h_{hs}$$

$$h_{col} > h_{hs}.$$